binomial distribution \((X \sim B(n, p))\)

A discrete probability distribution modeling the probability of a specific number of successes given a fixed number of trials and a fixed probability of success. A variable \(X \sim B(n, p)\) is said to be binomially distributed with \(n\) trials and a probability of success of \(p\). See figure BDD for details and figure BDC for function plots.

\[
\begin{align*}
\text{Figure BDC: } & \text{ binomial dist. probability functions; left: PMF of } X \sim B(13; 0.7) \text{ (solid) and } X \sim B(7; 0.5) \text{ (dashed); right: CDF of same distributions} \\
& \text{For example, the probability of } x \text{ out of } 7 \text{ children being girls follows the probability distribution } X \sim B(7, 0.5), \text{ assuming that the probability for a child being a girl is } P(\text{Girl}) = 0.5. \\
& \text{Given this distribution, the probability of three out of seven children being girls would be} \\
& P(X = 3) = f_B(3) = \binom{7}{3} \cdot 0.5^3 \cdot 0.5^{7-3} \approx 0.273 \\
& \text{where } \binom{n}{x} \text{ is the binomial coefficient.} \\
& \text{The probability } P(X \leq 3) \text{ of up to three out of seven children being girls would be:} \\
& F_B(3) = \sum_{w=0}^{3} f_B(w) = 0.5
\end{align*}
\]
### Binomial Distribution

**PMF**

\[ f(x) = \binom{n}{x} \cdot p^x \cdot q^{n-x} \]

**CDF**

\[ F(x) = \sum_{i=0}^{x} \binom{n}{i} \cdot p^i \cdot q^{n-i} \]

\[ F(x) = I_q(n - x, 1 + x) \]

**Statistic**

\( x \in \mathbb{N}_0, x \leq n \): number of successes

**Parameters**

- \( n \in \mathbb{N}_0 \): number of trials
- \( p \in [0, 1] \): probability of success
- \( q = 1 - p \) (probability of failure)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
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<tbody>
<tr>
<td>( \mu )</td>
<td>( np )</td>
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<tr>
<td>( \sigma^2 )</td>
<td>( npq )</td>
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**Skewness (\( \gamma_1 \))**

\[ \frac{q - p}{\sqrt{npq}} \]

**Approximations**

- \( N(np, npq) \) for \( np > 5, nq > 5 \)
- \( Poi(np) \) for \( n \geq 50, p < 0.1 \)

**Figure BDD:** Binomial distribution; \( I_x(a, b) \) is the regularized incomplete B (beta) function, a common method for computing the CDF of the distribution.