normal distribution (Gaussian distribution, $X \sim N(\mu, \sigma^2)$)
The most common *continuous probability distribution*. A *random variable* $X \sim N(\mu, \sigma^2)$ is normally distributed with mean μ and variance σ^2. The normal distribution plays a central role in statistics due to the *central limit theorem*, which basically says that the *average* of a large number of *iid* probability distributions, no matter of what type, converges toward the normal distribution.

The normal distribution is used to model a lot of natural phenomena, like body weight, intelligence quotients, etc. A measurement or *score*, like body weight or IQ, which follows a specific normal distribution, is called a "raw score". In order to find out how common a raw score is, it is converted to a z-*score* of the normalized *standard normal distribution* with mean $\mu = 0$ and variance $\sigma^2 = 1$.

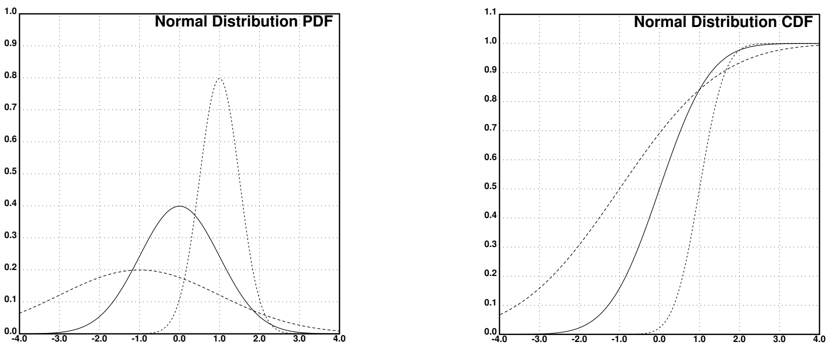

Figure **NOC:** normal distribution probability functions: left: PDF; right: CDF; Both panels: dashed line: $\mu = -1$, $\sigma^2 = 4$, solid line: $\mu = 0$, $\sigma^2 = 1$, dotted line: $\mu = 1$, $\sigma^2 = 0.25$

For example, given an IQ distribution $X \sim N(100, 15^2)$, a raw score of 117 would be correspond to the z-score

$$z = \frac{117 - 100}{\sqrt{15^2}} = \frac{17}{15} = 1.1\bar{3}$$

The *cumulative distribution function* (CDF) of the standard normal distribution can then be used to calculate the *quantile* of the z-score (and hence the quantile of the raw score). See figure NDD for the CDF.

Plotting the *probability density function* of the normal distribution exhibits a characteristic bell-shaped curve called the "bell curve" or "Gauss curve". See figure NOC.

$X \sim N(\mu, \sigma^2)$	
PDF	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
CDF	$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \int_{-\infty}^{x} e^{-\frac{(w-\mu)^2}{2\sigma^2}} dw$
	$F(x) = \frac{1}{2}\left(1 + \frac{erf(x-\mu)}{\sigma\sqrt{2}}\right)$
Statistic	$x \in \mathbf{R}$: raw score
Parameters	$\mu \in \mathbf{R}$: mean
	$\sigma^2 \in \mathbf{R}^+$: variance
μ	μ
σ^2	σ^2
Skewness (γ_1)	0

Figure **NDD:** normal distribution; erf is the Gauss error function