z-test for location
A hypothesis test used to find out whether a sample mean $\bar{x}$ differs significantly (→ significance) from a known mean $\mu_0$. It uses the standard normal distribution to compute the improbability of the observation $\bar{x}$. When the sample size is small ($n < 30$), the $t$-test for location should be used instead. The test statistic (“z-score”) used in the $z$-test is computed as follows:

$$z = \frac{\bar{x} - \mu_0}{\text{SE}} = \frac{\bar{x} - \mu_0}{\sqrt{s^2/n}}$$

where $s^2$ is the sample variance, $n$ is the sample size, and $\text{SE} = \sqrt{s^2/n}$ is the standard error of the sample.

The type of the $z$-test depends on its null hypothesis, $H_0$. When $H_0$ states $\bar{x} = \mu_0$, then the test is two-tailed, otherwise, when $H_0$ states either $\bar{x} \leq \mu_0$ or $\bar{x} \geq \mu_0$, it is one-tailed. The corresponding critical regions are shown in table ZTC.

<table>
<thead>
<tr>
<th>$H_0$</th>
<th>Critical Region</th>
<th>Tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x} = \mu_0$</td>
<td>$P(Z \leq -</td>
<td>z</td>
</tr>
<tr>
<td>$\bar{x} \leq \mu_0$</td>
<td>$P(Z \geq z) = \alpha$</td>
<td>Right</td>
</tr>
<tr>
<td>$\bar{x} \geq \mu_0$</td>
<td>$P(Z \leq z) = \alpha$</td>
<td>Left</td>
</tr>
</tbody>
</table>

Figure ZTC: critical regions of the $z$-test; $\alpha$ is the desired level of significance

The $z$-score (in $\sigma$; see standard deviation) indicates the improbability of the observed mean given the null hypothesis. It can be converted to a quantile using the cumulative distribution function $F_Z$ of the standard normal distribution.

Example: A training program is supposed to boost the IQ of its participants. The average IQ is $\mu_0 = 100$, the mean IQ taken from a sample of 50 participants is $\bar{x} = 105$ with a variance of $s^2 = 450$, and the chosen level of significance is $\alpha = 0.05$. The test score is then

$$z = \frac{105 - 100}{\sqrt{450/50}} = \frac{5}{3} \approx 1.667$$

Because in this case the null hypothesis $H_0$ would be that the IQ has not increased (i.e. $\bar{x} \leq \mu_0$), the corresponding critical region would start at 0.95. The quantile of 1.667 is $F_Z(1.667) \approx 0.952$,
so the result is (barely) within the critical region.

Figure **CRZ**: critical regions of the $z$-test Left: critical region in the left tail; middle: one critical region in each tail; right: critical region in the right tail