binomial distribution $(X \sim B(n, p))$

A *discrete probability distribution* modeling the *probability* of a specific number of *successes* given a fixed number of *trials* and a fixed probability of success. A variable $X \sim B(n, p)$ is said to be binomially distributed with n trials and a probability of success of p. See figure BDD for details and figure BDC for function plots.

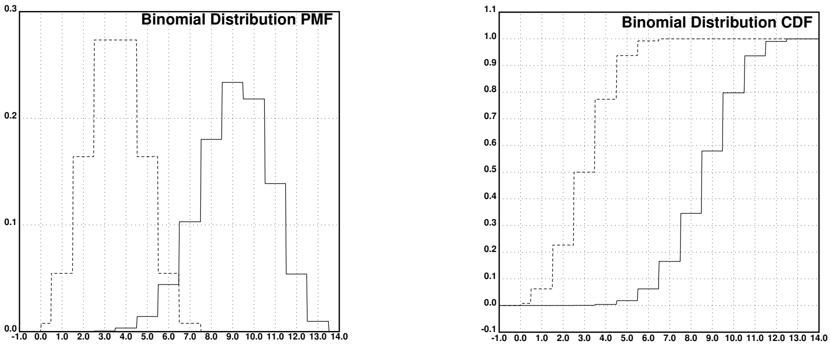

Figure **BDC:** binomial distr. probability functions; left: PMF of $X \sim B(13; 0.7)$ (solid) and $X \sim B(7; 0.5)$ (dashed); right: CDF of same distributions

For example, the probability of x out of 7 children being girls follows the probability distribution $X \sim B(7, 0.5)$, assuming that the probability for a child being a girl is $P(Girl) = 0.5$.

Given this distribution, the probability of three out of seven children being girls would be

$$P(X = 3) \; = \; f_B(3) \; = \; \binom{7}{3} \cdot 0.5^3 \cdot 0.5^{7-3} \; \approx \; 0.273$$

where $\binom{n}{x}$ is the *binomial coefficient*.

The probability $P(X \le 3)$ of up to three out of seven children being girls would be:

$$F_B(3) \; = \; \sum_{w=0}^{3} f_B(w) \; = \; 0.5$$

$X \sim B(n, p)$	
PMF	$f(x) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$
CDF	$F(x) = \sum_{i=0}^{x} \binom{n}{i} \cdot p^i \cdot q^{n-i}$
	$F(x) = I_q(n-x, 1+x)$
Statistic	$x \in \mathbf{N}_0, x \leq n$: number of successes
Parameters	$n \in \mathbf{N}_0$: number of trials
	$p \in [0, 1]$: probability of success
	q: $1 - p$ (probability of failure)
μ	np
σ^2	npq
Skewness (γ_1)	$\frac{q-p}{\sqrt{npq}}$
Approximations	$N(np, npq)$ for $np > 5, nq > 5$
	$Poi(np)$ for $n \geq 50$, $p < 0.1$

Figure **BDD:** binomial distribution; $I_x(a, b)$ is the regularized incomplete Β (beta) function, a common method for computing the CDF of the distribution